An Energy-Division Multiple Access Scheme

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Abstract - ***A new multiple access scheme based on energy discrimination is proposed. Such a scheme is based on the differences in the average power received by each user. It is particularly useful when no extra resources are available by means of existing schemes; moreover, it is useful when the system complexity must be kept very limited.***

Keywords: Multiple Access Protocols, Pulse Amplitude Modulation.

1 Introduction

Time-division multiple access (TDMA) and code-division multiple access (CDMA) are the main schemes introduced in the communication systems. Their advantages and disadvantages have been discussed in the classical literature [1,2,3,4].

In many applications several users may need to obtain multiple access using the same bandwidth, the same time interval and the same code. Such a scenario is interesting with reference to the case where the resources allocable with the classical scheme have been saturated. It is also interesting with reference to the case where several simple transceivers need to have a multiple access to a central network node but the transceivers need to be sufficiently simple so that a classical scheme is too difficult to be implemented. The latter scenario is relevant with reference to modern wireless personal area networks where terminals and base station (BS) need to interact without utilizing an existing infrastructure. It also applies to modern sensor networks where the communication infrastructure is not known *a priori*.

In this paper we propose a new multiple access scheme where several users simultaneously transmit with the same antipodal signaling scheme $\pm p(t)$. The information bit transmitted by each user is recovered by the receiver by using the differences in the average power of the signal received from each user. Such a scheme, which is named energy division multiple access (EDMA), is extremely simple to be implemented and allows one to achieve multiple access by a slight generalization of the power control.

2 The Proposed Model

We consider a set of N users transmitting to a BS on a single AWGN channel using simultaneously the same pulse for a simple binary antipodal PAM. We assume their transmissions are synchronous at the receiver antenna. Such an assumption is not needed in the considered scheme; it is introduced here in order to make clearer the basic idea on which the proposed scheme is founded. The baseband discrete-time signal at the BS, after matched filtering and sampling at the symbol rate, may be written as

$$y = \sum_{n=1}^{N} \alpha_n b_n + w \ , \tag{1}$$

where $b_n \in \{-1, +1\}$ is the bit of the nth user, $w \sim \mathcal{N}(0, \sigma^2)$ is the overall additive noise, and α_n is the signal level expressed as

$$\alpha_n = k \frac{\sqrt{\mathcal{E}_n}}{r_n} \ , \tag{2}$$

being $\mathcal{E}_n$ the energy transmitted by the nth user, r_n its distance from the BS, and k is a positive constant depending on the specific scenario. Without loss of generality we assume an ordering for the users determined by the amplitudes, i.e. $0 < \alpha_1 < \alpha_2 < \cdots < \alpha_N$. A more realistic channel modeling may take into account for the different channel attenuations encountered by the users. Such a model can be seen within the proposed framework provided that r_n represents

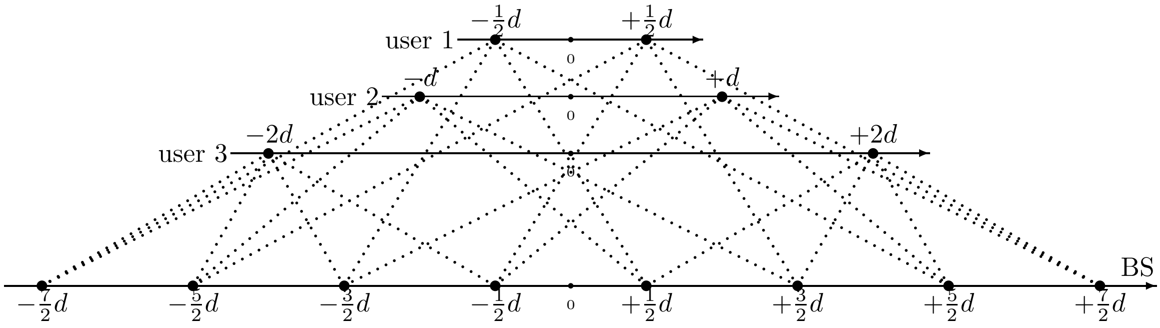

Figure 1: Amplitude in EDMA with 3 users.

the "equivalent" distance of the nth user, i.e., the r_n takes into account for the distance and the channel attenuation.

The N binary PAMs combine into a 2^N-ary PAM to the BS. In the following we analyze how to make the BS decode information from different users.

2.1 The Multiple Access Channel

When transmitting at same time, within the same band and with same code, some other form to distinguish different users is needed. Note that according to Eq. (1), all contributions from different users combine such that a scalar is obtained as sufficient statistics for the detection problem. The EDMA channel exists if we can decode each contribution correctly, that is the mapping from each combination of transmitted bits to a scalar is invertible. This can be easily shown to be equivalent to the following

Property of Separability:

$$\sum_{n=1}^{N} v_n \alpha_n \neq 0 \ , \tag{3}$$

for each combination with $v_n \in \{-1, 0, +1\}$ except the one with all coefficients null.

A sufficient condition for the *Property of Separability* is the following:

$$\alpha_n > \sum_{m=1}^{n-1} \alpha_m \ , \qquad n = 2, \ldots, N \ , \tag{4}$$

-*Proof*-

Each combination of transmitted bits has the form

$$y = \sum_{n=1}^{N} \alpha_n b_n \ . \tag{5}$$

Eq. (4) means that no euclidean distance between any two constellation points of the resulting PAM can be zero. Eq. (5) translates the Hamming distance for the binary transmitted vectors into euclidean distances on the real axis. Note that any two binary words can be at a Hamming distance of $1, 2, \ldots, N$. Consider all pairs of binary words whose Hamming distance is 1. The corresponding euclidean distances have the form

$$d_i^{(1)} = 2\,|\alpha_i| \ , \qquad i = 1, \ldots, N \ , \tag{6}$$

that is always not equal to zero because we have assumed that all amplitudes are positive. Consider all pairs whose Hamming distance is 2. The corresponding euclidean distance has the form

$$d^{(2)} = 2\,|\alpha_i \pm \alpha_j| \ , \qquad i, j = 1, \ldots, N \ , \tag{7}$$

that must not be zero, i.e. no pair of equal amplitudes is allowed. In general, for a Hamming distance equal to k, the corresponding euclidean distance has the form

$$d^{(k)} = 2\,|\alpha_{i_1} \pm \alpha_{i_2} \pm \cdots \pm \alpha_{i_k}| \ , \tag{8}$$

where the index set $\mathcal{I}^{(k)} = \{i_1, i_2, \ldots, i_k\}$ represents a subset of the users indexes. Without loss of generality, we can always assume that $\alpha_{i_1} > \alpha_{i_2} > \cdots > \alpha_{i_k}$. Separability imposes that all these distances are not null.

Let $\mathcal{I}_+^{(k)} \subseteq \mathcal{I}^{(k)}$ be the subset of indexes corresponding to the terms in Eq. (8) that contribute with a plus sign and, similarly, $\mathcal{I}_-^{(k)} \subseteq \mathcal{I}^{(k)}$ the set of amplitudes that contribute with a minus sign. Let $\mathcal{H}_k = \{1, 2, \ldots, i_k\}$ the set of index up to i_k. We can rewrite the distance $d^{(k)}$ by adding and subtracting all the terms $\{\alpha_j \ , \ j \in \mathcal{I}_+^{(k)} \cup \mathcal{H}_k - \mathcal{I}_-^{(k)}\}$

$$\begin{aligned} d^{(k)} &= 2\left|\alpha_{i_k} - \sum_{j\in\mathcal{I}_-^{(k)}} \alpha_j - \sum_{j\in\mathcal{H}_k - \mathcal{I}_-^{(k)}} \alpha_j \right. \\ &\quad \left. + \ 2\sum_{j\in\mathcal{I}_+^{(k)}} \alpha_j + \sum_{j\in\mathcal{H}_k-\mathcal{I}_-^{(k)}} \alpha_j \right| \\ &= 2\left|\alpha_{i_k} - \sum_{j\in\mathcal{H}_k} \alpha_j + K\right| . \end{aligned} \tag{9}$$

Note that $K = 2\sum_{m\in\mathcal{I}_+^{(k)}} \alpha_m + \sum_{j\in\mathcal{H}_k - \mathcal{I}^{(k)}} \alpha_j$ is a sum of positive terms and thus is always positive, unless no

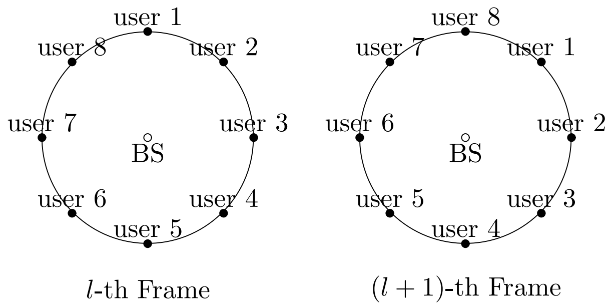

Figure 2: Rotation of energy assignment within the epoch.

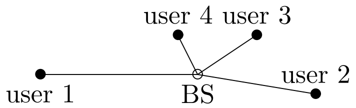

Figure 3: Energy assignment depending on the distance.

terms must be added, in which case $K = 0$. If we define

$$d_{i_k} \stackrel{\triangle}{=} \alpha_{i_k} - \sum_{j \in \mathcal{H}_k} \alpha_j \tag{10}$$

we can rewrite

$$d^{(k)} = 2\,|d_{i_k} + K| \ , \ \forall k = 1, \ldots, N \ , \ i_k = k, \ldots, N \ . \tag{11}$$

Eq. (4) is equivalent to

$$d_i > 0, \, i = 1, \ldots, N,$$

and if it holds we obtain that

$$d^{(k)} > 0$$

is always true. Hence all distances are always not equal to zero and no pair of constellation points collapses into the same point. Since this is true for any choice of k and i_k the theorem is proved.

It can be shown that the condition leading to an equally spaced 2^N-ary PAM, denoting with d the distance between two adjacent symbols, is

$$\alpha_n = \frac{d}{4} 2^n \ . \tag{12}$$

This also implies that no Gray coding is allowed. When such an equi-spaced condition holds, joint decoding can be easily achieved by successive cancellations provided that $d/\sigma \gg 1$.

The average energy of the system is (assuming $r_n = 1, \ k = 1$)

$$\mathcal{E}_{\rm av} = \frac{1}{N} \sum_{i=1}^{M} \mathcal{E}_i = \frac{2^{2N} - 1}{12N} d^2 \ . \tag{13}$$

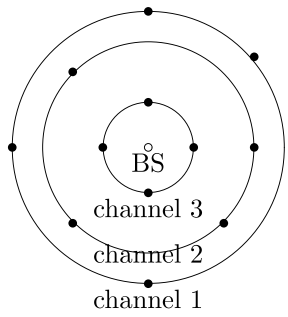

Figure 4: Channel assignment for Circle EDMA.

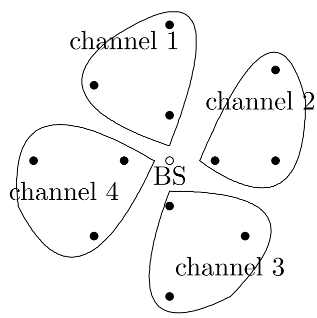

Figure 5: Channel assignment for Radial EDMA.

The average SNR is defined as follows:

$$\Gamma_{\rm av} = \frac{\mathcal{E}_{\rm av}}{2\sigma^2} \ . \tag{14}$$

2.2 Fairness

The proposed multiple-access scheme obtains different performances among the various users. In particular, two scenarios are noteworthy: the first one discussed in subsection 2.2.1 refers to the simple case where the channel attenuation is similar for each user and is synthetically depicted in Fig. 2 as the case where the distance of each user from the BS is constant; the second one, discussed in subsection 2.2.2, refers to the simple case where the channel attenuations of the users are significantly different and it is synthetically depicted in Fig. 3 as the case where the distance of each user from the BS is different.

2.2.1 Circle EDMA

On the basis of the differences of the energies required by each user and, consequently, of the different experienced Bit Error Rates (BER), a possible choice to render fair the system consists in periodically rotating the energy assignment to the users. We can identify 3 entities: *Time Slot* (TS), *Frame* and *Epoch*. They are defined as

- TS - the symbol interval, the time required for the bit transmission;

- Frame - the number of frames in which each user has a fixed assigned energy for transmission;
- Epoch - the number of successive frames in which each user experiences all the energy assignments, in a circular ordering.

A communication system may increase the number of handled users grouping along circles. Each circle implements EDMA on a different (TDMA, FDMA, CDMA) channel.

2.2.2 Radial EDMA

From Eq. (2)

$$\sqrt{\mathcal{E}_n} = \frac{d}{4k} r_n 2^n \ , \tag{15}$$

then the energy may be partially saved when numbering the users such that $r_1 > r_2 > \ldots > r_N$, so that the closest user (to the BS) generates larger received energy with a limited increase of the transmitted energy.

2.3 Hybrid Systems

The proposed multiple access scheme therefore needs to constitute groups of users that access the BS. Each group may be constituted of users transmitting with the same code. The overall system may provide circle grouping, as shown in Fig. 4, or radial grouping, as shown in Fig. 5. Therefore, particular attention needs to be payed to user grouping. For example, one may avoid that users with similar distances from the base station are included in the same subset of users whose multiple access is based on the proposed scheme. A successive refinement could be based on the capability of each user to switch among a few (e.g., 5) power levels. In such a case, simple modifications of the power control procedures are sufficient to guarantee multiple access among the different users.

3 Performance Analysis

The performance of the system may be evaluated in terms of joint error probability ($P_{\text{joint}}(e)$) and average error probability ($P_{\text{av}}(e)$). The former accounts for the rate of errors on the equivalent symbol on the BS-PAM and may be expressed as (see [2]):

$$P_{\text{joint}}(e) = \frac{2^N - 1}{2^N} \text{erfc}\left(\sqrt{\frac{3N}{2^{2N}-1}\Gamma_{\text{av}}}\right) \ . \tag{16}$$

The latter accounts for the average bit-error probability of the single user. A simple approximation (for large SNR) of the performance of the single user is derived. By assuming that errors only occur between adjacent points on the BS-ASK, the probability that n bits over N are wrong is

$$p_e(n, N) \approx 2^{N+1-n} q \ . \tag{17}$$

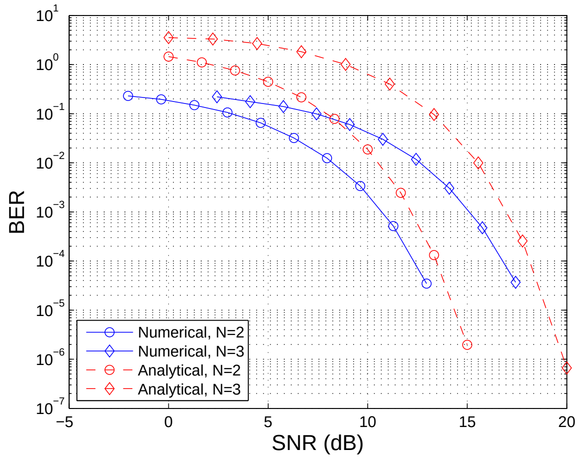

Figure 7: Performances obtained by numerical simulations and by analytical calculations.

Such a result is easily derived from the computation of the number of adjacent leaves in an M-ary tree of depth N whose distance is n, that is $2(M-1)M^{N-n}$ (see also Fig. 6), and the probability that a point on the BS-ASK is confused with one adjacent point, i.e.

$$q = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{3N}{2^{2N}-1}\Gamma_{\text{av}}}\right) - \frac{1}{2}\text{erfc}\left(\sqrt{\frac{27N}{2^{2N}-1}\Gamma_{\text{av}}}\right)$$

More precisely the probability that a point is confused with an adjacent point is q only if the latter it is an inner point, but for sake of simplicity we assume that it is true for all points. Finally we obtain

$$\begin{aligned} P_e &= \frac{1}{N}\sum_{n=1}^{N} n p_e(n, N) \\ &\approx \frac{4}{N}(2^{N+1} - N - 2)q \ . \end{aligned} \tag{18}$$

Computer simulations for systems with 2 and 3 binary users in the minimum energy configuration have been run using the software MATLAB. The analytical and numerical results are shown on Fig. 7.

4 Capacity Bounds

As for the capacity of the nth user for EDMA scheme, we consider an upper and lower bound obtained when no interference is assumed and when other users are approximated with a single Gaussian interference, respectively. The upper bound is

$$C_n = W \log_2\left(1 + \frac{3N2^{2(n-1)}}{2^{2N}-1}\Gamma_{\text{av}}\right) \ , \tag{19}$$

while the lower bound is

$$C_n = W \log_2\left(\frac{1}{1 - \frac{3N2^{2(n-1)}}{2^{2N}-1}\frac{\Gamma_{\text{av}}}{1+N\Gamma_{\text{av}}}}\right) \ . \tag{20}$$

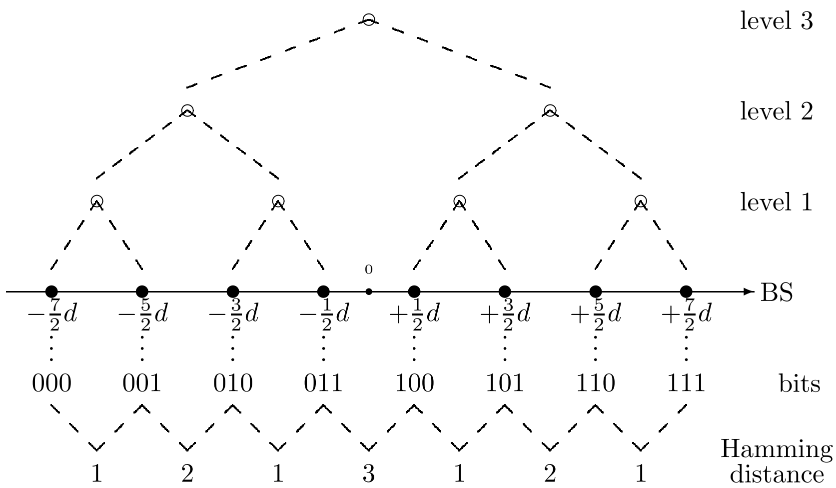

Figure 6: Tree structure for a system with 3 users.

5 Conclusions

Different users transmitting on the same channel with antipodal signaling can be separated on the basis of the differences in the received average power. We have introduced the condition that allows one to implement a linear complexity receiver at the expense of regulating the power of each user. Preliminary results show that such an idea is suitable and promising for scenarios where resources are scarce and system complexity must be kept to a minimum.

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